Does proper forcing give new instances of Scott's Problem?

Victoria Gitman
(CUNY – Graduate Center)

In 1962, Dana Scott defined the notion of Scott sets to characterize the properties of standard systems of models of PA. A Scott set is a Boolean algebra of subsets of the natural numbers that is closed under relative computability and paths through binary trees. The standard system of a model of PA is the collection of subsets of the natural numbers that are restrictions of the sets definable in the model. Every standard system is a Scott set. Scott showed that every countable Scott set is the standard system of some model of PA and twenty years later Knight and Nadel showed that every Scott set of size ω_1 is also the standard system of some model of PA. Thus CH implies that we have a complete characterization of standard systems. Nothing is known beyond ω_1 about the equivalence of Scott sets and standard systems.

I will use the forcing axiom PFA to provide new conditions under which Scott sets are standard systems of models of PA. By assuming that the Scott set X modulo the ideal of finite sets Fin is a proper poset, I will show how forcing with it allows you to construct a model whose standard system is X. This raises an unexplored open question of when is X/Fin proper.