18.311 Principles of Applied Mathematics.

Tue Th 9:30–11:00 in room 4–145

Textbooks: • **R. Haberman**, *Mathematical Models, Mechanical Vibrations, Population Dynamics and Traffic Flow*, SIAM. Covers (extensively) many of the topics in the course. Problems from this book will be assigned frequently.
• **C. C. Lin and L. Segal**, *Mathematics Applied to Deterministic Problems in the Natural Sciences*, SIAM. **An extremely good book to have.**

Class Notes: Class notes for some topics will be made available on the course web page.

References: The following books are also on reserve in the library:
• **F. Y. M. Wan**, *Mathematical Models and their Analysis*, Harper and Row. Covers some of the topics. **Problems from this book may be assigned occasionally.**
• **S. Strogatz**, *Nonlinear Dynamics and Chaos,* West-view press. This book covers the material in Part II of the course (and a whole lot more).
• **A. C. Fowler**, *Mathematical Models in the Applied Sciences,* Cambridge U. Press.
• **J. J. Stoker**, *Nonlinear Vibrations in Mechanical and Electrical Systems* , J. Wiley.
• **G. B. Whitham**, *Linear and Nonlinear Waves*, J. Wiley.
• **R. Haberman**, *Applied Partial Differential Equations With Fourier Series and Boundary Value Problems,* Prentice Hall.

Instructor: R. Rosales, room 2-337, x2784, rrr@math.mit.edu, Off. Hours: TBA.

TA: TBA.

Problem Sets: About one problem set per week (approx. 10 total). Each problem set will have two parts **(TO BE HANDED SEPARATELY)** — ”regular” and ”special.” The TA will grade the regular problems and the instructor the special ones.

GRADING: **Regular Problem Sets:** 30% of the grade.
Special Problem Sets: 70% of the grade.

Exams: There will be none.

E-mail: **Please read your mail regularly.** *Lots of important information will go via e-mail!* Check your mail this weekend. If you do not get a ‘test” e-mail from me, something is wrong. Warn me at my e-mail address above.

Policies: **Read the ”policies” write-up in the course web-page!**

IMPORTANT THINGS and RULES TO KEEP IN MIND. READ THIS:

1. **TYPE or PRINT** your name clearly on the Problem Sets.
2. *There will be stiff penalties for late homework and no make ups.*
3. The problem answers must be clear and easy to read, with proper explanations of the solution process. Use reasonably large characters (e.g. 12 points) and high contrast ink or pen. If your hand writing is not legible, type the answers. Use English, not mathematical jargon to explain your ideas. **Credit will be withdrawn for messy, hard to read and/or “out of the blue” answers.**
4. **Talk to me if you have a problem!** Don’t wait till it becomes unsolvable.
5. If you do not understand something in class: **STOP ME AND ASK** for a better explanation, please!

OUTLINE of the 18.311 LECTURES.

A rough idea of the topics to be covered follows. Some topics may be covered in more detail than this suggests, or the reverse. Some topics may be skipped completely and others may be included if needed. This is just to give you an idea of the flavor of the course.

- Computers and numerical issues. MatLab.
- Good and bad numerical schemes. Fourier Series and von Neumann stability analysis. Topics: consistency and stability of numerical schemes; von Neumann stability analysis; associated equation to a numerical scheme; short wave stability analysis; Discrete Fourier Transform (DFT); Fourier Series; Fourier Transform; Spectral Methods.

PART I. Some basic topics in Nonlinear Waves.

- Shock waves and hydraulic jumps. Description and various physical set ups where they occur: traffic flow, shallow water. What is a wave.
- Traffic flow (TF). Continuum hypothesis. Conservation and derivation of the mathematical model. Integral and differential forms. Other examples of systems where conservation is used to derive the model equations (in nonlinear elasticity, fluids, etc.)
- Linearization of equations of TF and solution. Meaning and interpretation. Solution of the fully nonlinear TF problem. Method of characteristics, graphical interpretation of the solution, wave breaking. Weak discontinuities, shock waves and rarefaction fans. Envelope of characteristics. Irreversibility in the model.
- Quasilinear First Order PDE's.
- Shock structure, diffusivity. Burger's equation. Explicit solution by the Cole-Hopf transformation. The heat equation. Derivation of the equation and solution. Application to the Burger's equation: Inviscid limit and Laplace's method. Dimensional analysis. Nonlinear diffusion and fronts.

PART II. Dynamical Systems.

- Basic notions of Dynamical Systems. Bifurcations, chaos, fractals, maps ...

PART III. Some of the following topics will be covered if time permits.

- Random walks, brownian motion, diffusion.
- Shallow water waves. Derivation of the equations. Linearization and solution. Radiation conditions. More on characteristics and shocks, now for the Shallow Water equations.
- Water waves. Derivation of the equations and linearization. Notions of dispersion and group speed. Weak nonlinearity and solitary waves. Perturbation expansions.
- Linear and nonlinear oscillations, relaxation. Phase plane methods and multiple scales. Applications to celestial mechanics and mechanical vibrations.
- Dynamical systems examples from mathematical biology and population dynamics.