

Due before class starts on April 29. Late homework will not be given any credit. Indicate on your report whether you have collaborated with others and whom you have collaborated with.

1. (20 points)

- Problem 3 in Problem Set 7.2 of the textbook (eigenvalues of Gauss-Seidel).
- Problem 10 in Problem Set 7.2 of the textbook (spectral radius of red-black Gauss-Seidel).

2. (50 points) Consider the domain Ω , as given in the figure. The boundary Γ consists of Dirichlet boundaries Γ_D (thin lines) and Neumann boundaries Γ_N (fat lines). Using finite differences, we would like to approximate the Poisson problem

$$\begin{cases} -\Delta u = 1 & \text{in } \Omega \\ u = f & \text{on } \Gamma_D \\ \frac{\partial u}{\partial n} = \frac{\partial f}{\partial n} & \text{on } \Gamma_N \end{cases},$$

where $f(x, y) = x^3y - xy^3 - \frac{1}{2}x^2$. Write a code that, for any given resolution parameter, yields

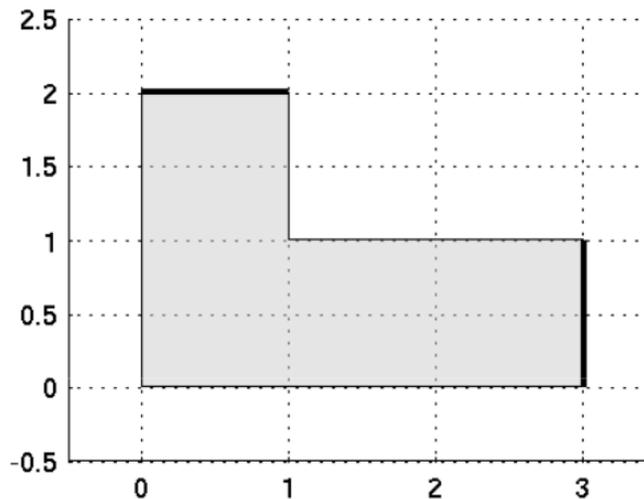


Figure 0.1: Domain.

a linear system that discretizes the above Poisson problem. Then choose a small resolution, a medium resolution and a high resolution, and (try to) solve the linear systems by

- Matlab backslash
- Jacobi iteration
- a simple multigrid method (V-cycle)

4. conjugate gradient method

Compare the approaches with respect to accuracy and run times. Up to which resolution can you go with the best method? Remarks:

- Use sparse matrices.
- Construct the matrix for interior points only, using lexicographic ordering. View either as a composition or a subtraction of two rectangles.
- Codes from the CSE web page can be used.
- Be aware of the difference between discretization error and errors in solving the linear system approximately.
- For a small test problem $Au = b$, the accuracy of a linear solver is best measured by the norm of the error $\|u_{true} - u_{approx}\|$ where u_{true} can be obtained by Matlab's backslash. However, for high resolutions, this error may not be accessible. In those cases, the residual $\|Au_{approx} - b\|$ can be used.
- The high resolution may cause some solution methods to fail. The medium resolution shall be chosen, so that all considered solution methods succeed to make a comparison.