

1. (a) Write down the 6×5 incidence matrix A_0 .
- (b) Find *all* solutions to $A_0^T y = 0$. (Remember the loops.)
- (c) Ground nodes 4 *and* 5: $x_4 = x_5 = 0$. Write out explicitly 9 equations for $x_1 x_2 x_3 y_1 \cdots y_6$ assuming $\underline{c = 2}$ in each edge.
- (d) Is the 9 by 9 matrix created in (c) positive definite? Why or why not? What is the definition of a positive definite matrix?
- (e) Find the matrix $A^T C A$ for this network.
How do you know it is positive definite?

2. To the top joint of a tree house,
my kids pinned a 45° flagpole:
- (a) Find the matrix A for this truss.
 - (b) Find all mechanisms x that solve $Ax = 0$.
How many are there? Draw a full set of independent mechanisms. Could the flagpole 5 have been better placed to give a completely stable truss?
 - (c) Suppose joint 4 is also fixed. Is this new truss statically determinate or indeterminate or unstable?
 - (d) With joint 4 fixed and all $c_i = 1$, what is A and what forces $f_H^1 f_V^1 f_H^2 f_V^2 f_H^3 f_V^3$ will produce $x_H^2 = 1$ and all other displacements = 0?

3. Given m equations $Ax = b$ in n ($n < m$) unknowns,

(a) The least squares solution \bar{x} is the vector that minimizes

_____ .

(b) This \bar{x} actually solves $A\bar{x} = b$ when b is in

_____ .

(c) The equation $A^T y = 0$ says geometrically that e is perpendicular to

_____ .

(d) There is a unique solution for e (True or False) because

_____ .

(e) The equation for \bar{x} is

_____ .

(f) This has a unique solution \bar{x} (always? any condition on A ?) if

_____ .

4. Instead of a line of springs we have a building fixed at the *bottom*.
- (a) Find the 3 equations that give the displacements x_1, x_2, x_3 in terms of the forces m_1g, m_2g, m_3g and the spring constants c_1, c_2, c_3 of the walls.
- (b) The matrix in $K \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} m_1g \\ m_2g \\ m_3g \end{bmatrix}$ can be factored into $K = LU$. What are L and U ?
 L has ones on the diagonal; c_1, c_2, c_3 are some positive numbers.
- (c) What are x_1, x_2, x_3 if $c_1 = c_2 = c_3 = 1$?