18.085 Prof. Strang   Quiz 3   December 11, 2002

Your name is: ______________________

Grading	1 (36)
	2 (36)
	3 (28)
Total	

In Questions 1 and 2, part (a) is checked by a later part—and Question 3 is not hard.

Thank you for taking 18.085. I appreciate the good homeworks and consistent effort.

I hope very much that you will find this subject useful.

(1a) GRAPH the 2π-periodic $f(x)$ defined between $-\pi$ and π by

$$f(x) = e^x \text{ for } -\pi \le x \le 0, \quad f(x) = e^{-x} \text{ for } 0 \le x \le \pi.$$

Find the Fourier coefficients c_k in $f(x) = \sum c_k e^{ikx}$ directly from the standard integral formula.

(1b) Draw the graphs of df/dx and d^2f/dx^2 between $-\pi$ and π. What rate of decay for the coefficients c_k?

(1c) Find the same coefficients c_k in a different way, starting from

$$-\frac{d^2 f}{dx^2} + f(x) = \text{ what function } R(x)?$$

With $f(x) = \sum c_k e^{ikx}$ take Fourier coefficients of both sides of this equation and solve for c_k.

(2a) Find the *cyclic* convolution $\boldsymbol{w} = \boldsymbol{u} * \boldsymbol{v}$ $(N = 4)$ of the vectors $\boldsymbol{u} = (1, 0, 1, 0)$ and $\boldsymbol{v} = (0, 1, 0, 1)$.

(2b) Find the discrete Fourier coefficients c_k and d_k (4-point DFT) of those vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively. From the c_k and d_k find the Fourier coefficients h_k of their convolution $\boldsymbol{w} = \boldsymbol{u} * \boldsymbol{v}$. How does this answer confirm that $\boldsymbol{w}$ in part (a) was correct??

(2c) For some 4-component vectors $\boldsymbol{z}$ the cyclic convolution with $\boldsymbol{u} = (1, 0, 1, 0)$ is $\boldsymbol{u} * \boldsymbol{z} = (0, 0, 0, 0)$. Describe the components of $\boldsymbol{z} = (z_0, z_1, z_2, z_3)$ ***and also*** its Fourier coefficients (C_0, C_1, C_2, C_3).

(3a) Find the Fourier integral transform $\widehat{f}(k)$ of the square wave $f(x)$ between -1 and 1:

$$f(x) = 1 \text{ for } 0 \le x \le 1, \qquad f(x) = -1 \text{ for } -1 \le x < 0, \qquad f(x) = 0 \text{ for } |x| > 1.$$

(3b) Find the Fourier integral transform $\widehat{D}$ of the *derivative* of that square wave. *Also* find the transform $\widehat{C}$ of the *convolution* of that square wave with its derivative. Factors of 2π are forgiven ...