

Your name is: _____

Grading 1
2
3
4

Total _____

- 1) (18 pts.) (1a) For which numbers q (allowing both positive and negative numbers) is this matrix positive definite?

$$K = \begin{bmatrix} 2 & q \\ q & 3 \end{bmatrix}.$$

- (1b) For q in this range find a function $f(x_1, x_2)$ that has its minimum at the solution to

$$K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}.$$

- (1c) If the columns of A are independent, what (if anything) can you conclude about the nullspace of A ?

2) (24 pts.) This system has six springs connected to three masses.

(2a) What are the balance equations between the spring forces y_1, \dots, y_6 (tension means $y > 0$) and the external gravity forces m_1g, m_2g, m_3g ?

(2b) Find the stiffness matrix $K = A^T C A$ for this system.

- (2c) Suppose all six spring constants are equal to $c = 5$ and all three masses are equal to $m = 6$. Find the displacements x_1, x_2, x_3 . You may use physical reasoning and/or linear algebra.
- (2d) Suppose a new seventh spring (with spring constant c_7) connects m_1 to m_3 . What will be the change in K ? (Which entries of K are changed and by how much?)

3) (36 pts.) The figure shows the 8 directed edges of a network with their conductances.

(3a) Find the incidence matrix A_0 . Explain without computing them why $A_0^T A_0$ and also $A_0 A_0^T$ are not positive definite.

(3b) A 1-amp source goes into node 1 and flows through the network to node 6 (grounded). Find the equations (DON'T SOLVE THEM) for the potentials x_1, x_2, x_3, x_4, x_5 .

- (3c) Based on symmetries of the network, which currents y_1, y_2, \dots, y_8 do you expect to be equal?
- (3d) Find as many independent solutions y_1, \dots, y_8 as possible to Kirchhoff's Current Law $A^T y = 0$ (with no source). Describe *all* solutions to this equation. Note: A and A_0 give the same solutions.
- (3e) Linear Algebra says that your y in nullspace of A^T is orthogonal to every vector $e = Ax$ in the column space of A . Write down the equation $y^T e = 0$ for one of your y 's in Problem 3d. What name does it usually have?
- (3f) Find one particular solution Y_1, \dots, Y_8 to $A^T y = f$ (with the 1-amp source term f included). Describe *all* solutions to this equation.

- 4) (**22 pts.**) This truss has six bars, two supports at $(0, 0)$ and $(3, 0)$ and four unsupported nodes with coordinates $(1, 2)$, $(2, 2)$, $(1, 1)$, $(2, 1)$.
- (4a) Find a complete set of independent mechanisms (displacement vectors x) that solve $Ax = 0$. *Draw them!!!*
- (4b) Add exactly enough bars to make this a stable statistically determinate truss (*draw your new truss*). If there are downward vertical forces $f_v = -1$ at all four unsupported nodes, find the forces in all of your bars.
- (4c) What are the horizontal and vertical reaction forces at the two supports? First give the force balance equations at those supports.