April 16, 2008

**18.702 Problem Set 7**

due wednesday, April 23

1. Determine the irreducible polynomial for $\gamma = \sqrt{3} + \sqrt{5}$ over each of the fields $\mathbb{Q}$, $\mathbb{Q}(\sqrt{5})$, $\mathbb{Q}(\sqrt{10})$, and $\mathbb{Q}(\sqrt{15})$.

2. Let $\zeta_n = e^{2\pi i/n}$. Determine the irreducible polynomial over $\mathbb{Q}$ of $\zeta_6$, $\zeta_9$, and $\zeta_{12}$.

3. Let $\alpha$ and $\beta$ be complex numbers. Prove that if $\alpha + \beta$ and $\alpha\beta$ are algebraic numbers, then so are $\alpha$ and $\beta$.

4. Prove in two ways that the regular pentagon can be constructed by ruler and compass:
(a) by field theory, and (b) by finding an explicit construction.

5. For which fields $F$ and which primes $p$ does the polynomail $x^p - x$ have a multiple root?

6. Factor $x^9 - x$ and $x^{27} - x$ over the field $\mathbb{F}_3$.

7. Prove that $\sqrt[3]{5}$ is not in the field $\mathbb{Q}(\sqrt[3]{2})$.

8. Prove that every element of $GL_2(\mathbb{Z})$ of finite order has order $1, 2, 3, 4$, or $6$.