

18.702 Problem Set 6

due friday, April 11

1. Chapter 12, Problem 1.6.

2. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

be an matrix with entries in a field F . The matrix $tI - A$ can be diagonalized using elementary row and column operations in the polynomial ring $F[t]$. What diagonal matrices might be obtained?

3. Chapter 12, Problem 4.9.

4. Using elementary row and column operations in the ring $\mathbb{Z}[i]$ of Gauss integers, diagonalize the matrix

$$\begin{pmatrix} 3 & 2+i \\ 2-i & 9 \end{pmatrix}.$$

5. Let R denote the ring of Gauss integers, and let W be the R -submodule of $V = R^2$ generated by the columns of the matrix

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix},$$

with $\alpha, \beta, \gamma, \delta \in R$. Find a simple way to determine the index of W in V .

6. Chapter 12, Problem 5.1.

7. Chapter 12, Problem 6.3a,b .

8. Chapter 12, Misc. Problem 2.