Erratum to: Fukaya categories and Picard-Lefschetz theory, Lemma 2.1

Lemma 2.1 as stated is obviously incorrect. Take an A_∞-algebra $\mathcal{A}$ which is acyclic (has vanishing cohomology in all degrees). Then, $\mathcal{A}$ is tautologically cohomologically unital. However, one can easily find examples in which the multiplication cannot be transformed into a strictly unital one by a formal diffeomorphism (for instance, one can take any nontrivial acyclic chain complex which has no nonzero cocycles in degree 0, and set the multiplications $\mu^d_{\mathcal{A}}$, $d \geq 2$, to be equal to zero).

The issue happens at the start of the proof. Keeping to the case of A_∞-algebras for simplicity, if $\mathcal{A}$ is acyclic, one cannot in general represent the multiplication by a chain map $\mu^2_{\mathcal{A}} : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$ such that $(-1)^{|a|}\mu^2_{\mathcal{A}}(e,a) = a = \mu^2_{\mathcal{A}}(a,e)$ (in the example above, one necessarily needs to set $e = 0$, and then the condition becomes impossible to satisfy unless $\mathcal{A} = 0$). The correct version of the statement, which sidesteps this issue, is:

Lemma 2.1. *Let $\mathcal{A}$ be a cohomologically unital A_∞-category, such that for all objects X, the complex $hom_{\mathcal{A}}(X,X)$ is either zero or has nontrivial cohomology. Then there is a formal diffeomorphism Φ with $\Phi^1 = Id$, such that the modified A_∞-structure $\Phi_*\mathcal{A}$ is strictly unital.*

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