ERRATUM TO "SOME SPECULATIONS ON PAIRS-OF-PANTS DECOMPOSITIONS AND FUKAYA CATEGORIES"

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Abstract. We correct an error in [1]. This Erratum will not be published. I would like to thank Umut Varolgunes for pointing out the mistake to me.

The isomorphism in [1, Equation (3.4)] is incorrect. A correct statement is this:

$$CF^0(L,L;H) \cong \mathcal{O}_B^{unif}(U), \tag{1}$$

where $\mathcal{O}_B^{unif}(U)$ is the ring of non-archimedean power series which are uniformly q-adically convergent in U, in a sense to be explained below. On both sides of (1) we are considering formal power series

$$f(b) = a_1 b^{v_1} + a_2 b^{v_2} + \cdots \quad a_i \in \Lambda_q,\ v_i \in \mathbb{Z}^n. \tag{2}$$

Take a convex function $H : U \to \mathbb{R}$ as in [1, p. 419]. Note that by construction, H is bounded. Let $K : \mathbb{R}^n \to \mathbb{R}$ be its Legendre transform:

$$K(v) = H(b) - v \cdot b, \quad \text{where } b \in U \text{ is the unique point such that } dH_b = v; \tag{3}$$

or equivalently,

$$K(v) = \min_{b \in U}(H(b) - v \cdot b). \tag{4}$$

On the left hand side of (1), the definition of the Floer complex has the effect of imposing the following condition on the coefficients of (2):

$$\lim_{i\to\infty} \mathrm{val}(a_i) + K(v_i) = \infty, \tag{5}$$

where $\mathrm{val}(a_i) \in \mathbb{R} \cup \{\infty\}$ is the q-adic valuation. By "uniformly q-adically convergent in U", we mean the following:

$$\lim_{i\to\infty} \mathrm{val}(a_i) - v_i \cdot b = \infty \quad \text{uniformly with respect to } b \in U. \tag{6}$$

Of course, a sequence of functions converges to ∞ uniformly if and only if the infima of those functions converge to ∞. Hence, given (4), (5) and (6) are equivalent.

For comparison with the original statement [1, Equation (3.4)], note that $\mathcal{O}_B^{unif}(U)$ is a sub-ring of $\mathcal{O}_B(U)$, but not equal to it. For instance, take $U = (0,1)$, and consider

$$1 + qx + q^2x^2 + \cdots \tag{7}$$

The functions $i(1-b)$ converge to ∞ pointwise for $b \in (0,1)$, but not uniformly so. Hence, (7) belongs to $\mathcal{O}_B(U)$, but not to $\mathcal{O}_B^{unif}(U)$.

References

[1] P. Seidel. Some speculations on pairs-of-pants decompositions and Fukaya categories. In *Surveys in differential geometry. Vol. XVII*, volume 17 of *Surv. Differ. Geom.*, pages 411–425. Int. Press, Boston, MA, 2012.