

# ERRATA

## for *Enumerative Combinatorics*, vol. 1, 2nd ed., 2012

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- page 5, line 9–. Insert “a” after “is”.
- page 23, line 6. Insert “a” after “is”.
- page 24, line 2. Change  $c_i$  to  $c_1$ .
- page 34, line 15. It should be stated that for this second way,  $d + 1$  cannot be inserted at the end even though  $d \notin D(w)$ .
- page 38, line 17. Change “such letter appears in exactly one compartment” to “compartment contains exactly one such letter”.
- page 38, line 18. Change “does” to “do”.
- page 40, line 15. The intent of the parenthetical comment was to write the summation as

$$\sum_{g(4) \geq g(6)} \sum_{g(6) \geq g(3)} \dots \sum_{g(5) \geq 0} q^{g(4) + \dots + g(5)}.$$

We are actually first summing on  $g(5) \geq 0$ , then  $g(1) \geq g(5)$ , etc.

- page 40, line 4–. Insert  $\sum_{m \geq 0}$  before  $\sum_{w \in \mathfrak{S}_d}$ .
- page 43, line 4. Change “relection” to “reflection”.
- page 43, line 14. Change  $i \geq 0$  to  $i \geq 1$ .
- page 45, line 1. Change  $w$  to  $T(w)$ .
- page 46, line 13–. Remove bracket at end of line.
- page 47, line 11. Change  $n \geq 2$  to  $n \geq 1$ .
- page 49, lines 20–22. The two sentence beginning “Figure 1.10 ...” and “Let  $f(n)$  denote ...” should be interchanged, since  $f(n)$  is used in the first of these sentences but defined in the second.

- page 50 , line 4. Change “tree” to “trees”.
- page 53, line 1–. Change  $n$  to  $n - 1$ .
- page 54, line 8. Change “number” to “numbers”.
- page 61, Figure 1.15. Change  $w = 1212112$  to  $w = 12121121$ .
- page 62, equation (1.78). Change  $j \in \mathcal{S}_i$  to  $j \in S_i$ .
- page 62, line 13–. Change  $\mathcal{S}_j$  to  $S_i$ .
- page 65, line 3. Perhaps it is misleading to say that the bijection is “easy to check”.
- page 65, line 7. Change proposition to Proposition.
- page 66, line 13. Change  $F(0, q)$  to  $F(x, 0)$ .
- page 68, line 17–. Change “ $k$  of the  $\gamma_i$ ’s” to “ $k$   $\gamma_i$ ’s”.
- page 70, lines 3 and 5. Replace “replacing” with “placing”.
- page 71, line 2–. Change “then then” to “then the”.
- page 76, line 2. Change “ $x$  to  $-x$ ” to “ $t$  to  $-x$ ”.
- page 77, line 5–. Change “ $f(0), f(0)$ ” to “ $f(0), f(1)$ ”.
- page 78, line 5. Note that in the formula  $\Delta(x)_n = n(x)_{n-1}$ ,  $\Delta$  is acting on  $x$ , not on  $n$  as before.
- page 85, Theorem 1.10.4, line 3. Change  $\text{GL}(n, q)$  to  $\text{GL}(m, q)$ .
- page 88, line 11–. Change  $m_k$  to  $m_k - 1$ .
- page 88, line 13–. Change  $v_{ij}$  to  $v_{1j}$ .
- page 89, line 3–. Change 1.8.6(c) to 1.8.6(a).
- page 92, line 5. Change  $f \neq 0$  to  $f \neq z$ .
- page 92, line 7. Insert the exponent  $-1$  at the end of this line.
- page 94, line 15. Change  $\psi(F)$  to  $\psi(A)$ .

- Lemma 1.10.14, line 3. The case  $n = 0$  is unnecessary since the formula for  $a_n$  is valid for  $n = 0$ .
- Proof of Lemma 1.10.14. Bruce Sagan notes that there is a simple combinatorial proof that  $a_n = (q - 1)^{n-1} - a_{n-1}$ . The proof of the lemma follows by induction.
- page 96, line 3-. Change  $\#\Gamma_{\text{id}}$  to  $\#(\text{GL}_0(n, q) \cap \Gamma_{\text{id}})$ .
- page 98, line 7. Change “in the monograph” to “is the monograph”.
- page 98, line 20. Change Rodriques to Rodrigues.
- page 98, line 16-. Change “papers [1.23][1.24]” to “paper [1.24]”.
- page 104, Exercise 3.3(e). We should assume  $n \geq 1$ .
- page 109, Exercise 35(b,c,d). For the history of these results, see MathOverflow 63561.
- page 111, line 2. Change “second” to “third”.
- page 111, Exercise 49, line 1. Change (1.38) to (1.37).
- page 112, Exercise 54(a), line 1. Change  $A(2n+1, n+1)$  to  $A(2n+1, n)$ .
- page 112, Exercise 55. One should compute  $\text{maj}(w^r)$  in terms of both  $\text{maj}(w)$  and  $\text{des}(w)$ , not just  $\text{maj}(w)$ .
- page 112, Exercise 55, line 2. Delete “, respectively”, since  $\text{maj}(w^r)$  is expressed in terms of both  $\text{des}(w)$  and  $\text{maj}(w)$ .
- page 113, Exercise 1.58(a), line 2. Change “and” to “can”.
- page 114, Exercise 61(b). It would be better to use a different letter than  $u$ , since  $u$  has different meaning in part (a).
- page 114, Exercise 61(c). Although there is technically no error, the terminology in parts (a) and (c) is inconsistent. In (a) the peaks and valleys are the terms  $w_i$ , while in (c) the double falls are the indices  $i$ .
- page 114, Exercise 62(b), line 2. Change  $k$  to  $i$  (twice) (in order to be consistent with the notation of the solution on page 160).

- page 115, Exercise 70(b), line 3. Change  $(1, 0)$  to  $(0, 1)$ , and change “i.e,” to “i.e.,”.
- page 120, Exercise 94, second bullet. Change “Then” to “The”.
- page 121, Exercise 96(c), line 3. Change 4.4.1.1 to 4.1.1.
- page 121, Exercise 98. Strictly speaking, when we put  $q = \zeta$  in the definition of  $\begin{bmatrix} n \\ k \end{bmatrix}$  on page 55, we get  $0/0$ . To be precise we should write  $\lim_{q \rightarrow \zeta} \begin{bmatrix} n \\ k \end{bmatrix}$ .
- page 123, Exercise 116(b), line 2. Change distribuiton to distribution.
- page 125, line 3. Change  $\Delta^j g_k(-n)$  to  $\Delta^j g_k(-n)|_{n=0}$ .
- page 126, line 2. Change  $1 \leq i < j \leq n$  to “ $1 \leq i \leq j \leq n$ , excluding  $(i, j) = (1, n)$ ”.
- page 126, Exercise 128(d), line 7. Change  $\frac{2}{1+x}$  to  $\frac{2x}{1+x}$ .
- page 126, Exercise 129(a), line 3. Add  $\sum_{k \geq 0}$  after  $\sum_{n \geq 1}$ .
- page 127, Exercise 135, line 2. Change “ $n$  even” to “ $n$  even,  $n \neq 0$ ”.
- page 132, Exercise 161(a), lines 2 and 3. Change  $\Phi_{k,0}$  to  $\Psi_{k,0}$  (in order to be consistent with the previous exercise and to avoid possible confusion with the cyclotomic polynomial  $\Phi_k(x)$ ). It would also be better notationally to write  $F_k(x)$  instead of  $\Psi_{k,0}(x)$  on line 3.
- page 136, line 1–. Change  $q^{n-1}$  to  $q^{k-1}$ .
- page 137, Exercise 184(a), line 5. Change “polynomials  $f(x) = \beta$  for  $0 \neq \beta \in \mathbb{F}_q$  are” to “polynomial  $f(x) = 1$  is”.
- page 137, Exercise 184(b), line 1. To be consistent with (a), change  $f(n, q)$  to  $f_q(n)$ .
- page 138, Exercise 186(a), line 3. Insert “of Exercise 1.185” after “(b)”.
- page 138, equation (1.148). Change  $q^{m+n-1}$  to  $q^{m+n-1}(q-1)$ .
- page 139, Exercise 193, line 2. Change  $\gamma_n$  to  $\gamma(n)$ .

- page 140, Exercise 199, line 4. The condition (only relevant in characteristic 2) should be that  $A = -A^t$  and  $A$  has 0's on the main diagonal. Some people will take this to be the definition of “skew-symmetric,” while others will call these matrices “alternating.”
- page 140, Exercise 198(b), last displayed equation. Change  $(q^{2m-1})$  to  $(q^{2m-1} - 1)$  (twice).
- page 148, Exercise 56, line 3. Change  $=$  to  $\subseteq$ .
- page 150, Exercise 22(b), line 6. Change  $x^{2i+1}$  to  $x^{2i-1}$ .
- page 151, Exercise 25, line 3–. Change “ $r$  of these parts” to “ $s$  of these parts”.
- page 151, Exercise 25, line 3–. Change  $\binom{r+s}{r}$  to  $\binom{r+s}{s}$ . Of course these two binomial coefficients are equal, but it is more logical to use  $\binom{r+s}{s}$ .
- page 152, Exercise 35(f), lines 7–8. In the sentence “Replace ...”, change the two 1's to 2's, and change the 2 to 1.
- page 152, Exercise 35(f), line 9. Change each 1 to 2 and each 2 to 1.
- page 153, Exercise 36, line 2. Change  $f_k(1)$  to  $f_1(k)$  (twice).
- page 155, Exercise 43, line 8–. Change  $j$  to  $n - j$ .
- page 156, line 2. Change *Combinatorica* to *Combinatoria*.
- page 156, Exercise 49, line 3. Change the comma at the end of the line to a period.
- page 156, Exercise 49, line 4. Delete this line.
- page 156, Exercise 49, line 5. Delete **b.** at the beginning of the line.
- page 156, line 9–. Change  $f(x) = 0$  to  $(1 - x)^{-d-1}A_d(x) = 0$ .
- page 159, line 8–. Change  $F - bx$  to  $F - x$ .
- page 160, line 12. Change  $g(n + 1) +$  to  $g(n + 1) =$ .
- page 160, Exercise 62. Change **a.**, **b.**, **c.** to **b.**, **c.**, **d.**.

- page 160, lines 5–6. Perhaps the sentence “This bijection . . . and so on.” would be clearer if it were replaced with “*Hint.* This bijection has the property that  $\max\{a_1, \dots, a_n\} = \text{des}(\rho(w)^{-1}) + 1$ .”
- page 163, Exercise 80. For the convoluted history of this result, see R. Gilbert, A Fine rediscovery, *Amer. Math. Monthly* **122** (2015), 322–331. In particular, the result is actually first due to N. J. Fine in 1959.
- page 164, Exercise 83, lines 7–8. Change  $(a, a)$  to  $(u, u)$  and  $(a + 1, a)$  to  $(u + 1, u)$  (since  $a$  has already been used as an indeterminate).
- page 164, Exercise 83, line 4–. Change the second denominator factor  $(1 - a^j b^j c^{j-1} d^{j-1})$  to  $(1 - a^j b^{j-1} c^j d^{j-1})$ .
- page 165, Exercise 89, line 7. Change  $n$  to  $k$  (twice).
- page 166, line 1. While this sentence is correct, there is direct agreement with equation (1.88) if one substitutes  $-q^{-1/2}$  rather than  $-q^{1/2}$  for  $x$ .
- page 166, Exercise 91(d), line 4. Replace this line with

$$1 + \sum_{n \geq 1} (-1)^n (x^n - x^{-n-1}) q^{\binom{n}{2}}.$$

- page 166, Exercise 92. This identity is actually due to I. Gessel and D. Stanton, *Trans. Amer. Math. Soc.* **277** (1983), 173–201 (equations (7.13) and (7.15)).
- page 168, Exercise 104, line 1. Notation is confusing! Change  $f(x)$  to  $F(x)$ .
- page 168, Exercise 104, line 1. Change  $i^2 = 1$  to  $i^2 = -1$  and  $f(x)$  to  $F(x)$ .
- page 168, Exercise 104, line 2. Change the last four  $f$ ’s to  $F$ .
- page 168, Exercise 104, line 4. Change the four lines with a left brace to

$$= \begin{cases} \frac{1}{4} (10^n + (-1)^k 2^{2k+1}), & n = 4k \\ \frac{1}{4} (10^n + (-1)^k 2^{2k+1}), & n = 4k + 1 \\ \frac{1}{4} 10^n, & n = 4k + 2 \\ \frac{1}{4} (10^n - (-1)^k 2^{2k+2}), & n = 4k + 3. \end{cases}$$

- page 169, Exercise 108, line 3–. Change “in papers” to “in the papers”.
- page 169, Exercise 109(a), line 4. Change “is bijection” to “is a bijection”.
- page 169, Exercise 111. The first sentence may be unclear. We are counting the number of ways to choose a  $k$ -element subset  $S$  of  $[n]$  and then partition  $S$  into  $j$  intervals.
- page 170, Exercise 117. A simpler proof was suggested by Bruce Sagan. Let the cycle containing 1 be  $(1, a_2, \dots, a_k)$ . There are  $(n-1)(n-2) \cdots (n-k+1)$  ways to choose  $a_2, \dots, a_k$  and then  $(n-k)!$  ways to permute the elements not in this cycle, etc. Perhaps the difficulty rating of this exercise should be [2].
- page 170, Exercise 118(b). As pointed out by Bruce Sagan, there is a simple solution analogous to the solution above for Exercise 117.
- page 171, line 4. The index entries for these four names is missing.
- page 171, line 6. Insert a space between “Let” and  $v$ .
- page 172, line 6. Change  $(-1)^j b_{k-j}$  to  $(-1)^j \binom{-n}{j} b_{k-j}$ .
- page 173, Exercise 129. Change the parts **a** and **b** to **b** and **c**.
- page 173, Exercise 134, line 2. Change the period after Petersen to a comma.
- page 176, Exercise 146. Bruce Sagan points out that this exercise can also be done directly from (1.59) and a simple Inclusion-Exclusion argument.
- page 176, line 7–. Change “left subtree” to “right subtree”.
- page 179, line 4. Change  $F_1\left(\frac{1}{1+x}\right)$  to  $F_1\left(\frac{x}{1+x}\right)$ .
- page 179, line 5. Change  $G\left(\frac{1}{1+x}\right)$  to  $G\left(\frac{x}{1+x}\right)$ .
- page 180, Exercise 161(b). Change every appearance of  $H$  in this solution to  $F$ .

- page 181 line 5. The two square roots should be preceded by  $\pm$  since their signs have yet to be determined.
- page 181, Exercise 164, line 1. Change  $x = 0$  to  $y = 0$ .
- page 181, line 4-. Change  $x$  to  $y$ .
- page 182 Insert after “ $x + p_2x^2 + \dots$ .” the following sentence:  
Then we need to show that  $p_{2n} \in I := \langle p_2, \dots, p_{2n-1} \rangle$ , the ideal of the polynomial ring  $K[a_2, a_3, \dots]$  generated by  $p_1, \dots, p_{2n-1}$ .  
The sentence after this one should begin a new paragraph.
- page 183, line 5. Change  $A(-A(-x)$  to  $A(-A(-x))$ .
- page 183, Exercise 168. The parts **b.**, **c.**, **d.**, **e.**, **f.** should be **d.**, **f.**, **g.**, **i.**, **j.**, respectively.
- page 183, line 11-. Change  $n \geq$  to  $n \geq 0$ .
- page 184, line 1. Change  $xy^2$  to  $x^2y$ .
- page 186, lines 4–5. Change  $D$  to  $E$  (twice), since  $D$  has already been used.
- page 186, Exercise 164. Change the parts **a** and **b** to **b** and **c**.
- page 186, Exercise 185, line 6. Insert  $\prod_{d \geq 1}$  after  $=$ .
- page 188, Exercise 188, line 4. Change  $\mathbb{F}_q$  to  $\mathbb{F}_q^n$  (twice).
- page 189, line 18-. Insert the  $q$ -binomial coefficient  $\binom{n}{r}_q$  after  $\sum_{r=1}^n$ .
- page 190, Exercise 1.190(b), line 3. Replace this line with
$$\omega^*(n, q) = q^n - q^m - q^{m-1} - q^{m-2} - \dots - q^{\lfloor n/3 \rfloor} + O(q^{\lfloor n/3 \rfloor - 1}).$$
- page 191, third solution to Exercise 191. Change  $\Phi$  to  $\Phi_M$  (four times) for notational consistency.
- page 192, lines 3 and 8. Change  $\gamma_n$  to  $\gamma(n)$ .



- page 192, line 3–. insert “/q” before the period.
- page 193, line 2. Change  $\gamma_{n-k}(q)$  to  $\gamma(n-k, q)$ .
- page 195, line 9–. Change “example” to “examples”.
- page 196, equation (2.5). Change  $Y \supseteq T$  to  $Y \supseteq \emptyset$  or to just  $Y$ .
- page 198, line 3. Change  $f = (T)$  to  $f_{=}(T)$ .
- page 198, line 2. This line should read:

function  $f_{=}$  satisfies  $f_{=}(T) = f_{=}(T')$  whenever  $\#T = \#T'$ . Thus also  $f_{\geq}(T)$

- page 199, line 11–. Change “i set” to “i-set”.
- page 199, line 3–. Change  $S - T$  to  $S_n - T$ .
- page 200, line 8. Change  $j \geq i$  to  $j \leq i$ .
- page 203, line 6. Insert ) before }.
- page 203, line 13. Change “nonattacking” to “nonattacking”.
- page 203, line 4–. Change “is” to “in”.
- page 205, line 1. Change “revisted” to “revisited”.
- page 205, line 2–. Change  $m - k - 1$  to  $m - k + 1$ .
- page 206, line 9. Change “suggest” to “suggests”.
- page 207, line 4–. Change  $x - s_1$  to  $x + s_1$ .
- page 208, line 10–. Change “ $a_3$ ’s” to “ $a_3$  3’s”.
- page 210, line 9. Insert “one for 121,” after “one for 112,”.
- page 210, line 9–. Insert = between  $d(n)$  and  $\#D_n$ .
- page 212, line 3. Insert a period after  $b_1 > a_1$ , and raise the end of proof symbol to the same line as the period.

- pages 212–213. There is a simpler involution proof of (2.30) (equivalently, (2.5)). See pages 46–47 of B. Sagan, *The Art of Counting*.
- page 212, line 9–. Change  $T \subseteq S$  to  $Y \subseteq S$ .
- page 216, line 5–. Insert  $)$  after  $x_{\gamma_i}$ .
- page 217, l. 14–. Change  $v$  to  $(x, y)$ .
- page 217, l. 12–. Change  $v$  to  $(x, y)$ .
- page 222, Exercise 10(b), lines 1 and 2. Change  $E(n)/n!$  to  $E(n)/n^n$  (twice).
- page 222, Exercise 11. This exercise is stated incorrectly. The correct statement is the following.

- (a) [3–] Fix a subset  $S$  of  $[n - 1]$ . Let  $a_1, \dots, a_k$  be the sizes, from left-to-right, of the maximal subsets of  $S$  that are sequences of consecutive integers. For instance, if  $S = \{2, 5, 6, 7, 9, 12, 13\}$  then  $(a_1, a_2, a_3, a_4) = (1, 3, 1, 2)$ . Write  $\varphi(S) = (a_1, \dots, a_k)$ . Let  $M(a_1, \dots, a_k)$  denote the number of permutations in  $\mathfrak{S}_n$  with descent set  $S$  having  $n - \#S$  fixed points (the maximum possible). Show that  $M(a_1, \dots, a_k)$  is the coefficient of  $x_1^{a_1} \cdots x_k^{a_k}$  in the expansion of

$$\frac{1}{(1 + x_1)(1 + x_2) \cdots (1 + x_k)(1 - x_1 - x_2 - \cdots - x_k)}. \quad (1)$$

*Example.* The coefficient of  $x_1^{2m}$  is 1. If  $n = 2m + 1$  then this corresponds to the permutation  $2m + 1, 2m, \dots, 1$  with one fixed point. The coefficient of  $x_1^{2m-1}$  is 0, since there is no permutation in  $\mathfrak{S}_{2m}$  with descent set  $[2m - 1]$  and one fixed point.

- (b) [1+] Show that  $M(a_1, \dots, a_k)$  does not depend on the order of the numbers  $a_1, \dots, a_k$ .
- (c) [2+] Show that the coefficient of  $x_1 x_2 \cdots x_k$  in the expansion of equation (1) is  $D_k$ , the number of derangements in  $\mathfrak{S}_k$ . Deduce that the number of alternating permutations in  $\mathfrak{S}_{2m}$  (or in  $\mathfrak{S}_{2m+1}$ ) with  $m$  fixed points is  $D_m$ .

- page 223, line 4. Change  $[n]$  to  $[k]$ .
- page 223, Exercise 13(b), line 4. Change  $S' \supseteq S$  to  $S' \subseteq S$ .
- page 224, Exercise 14(a), line 8. Insert  $+36 \cdot 5^n$  after  $-36 \cdot 6^n$ .
- page 225, Exercise 18, line 3. To be consistent with Exercise 17, change  $w(i+j)$  to  $a_{i+j}$  and  $w(i)$  to  $a_i$ .
- page 227, line 3–. On the left-hand side of the formula, remove one  $\sum_{i \geq 0}$  and change  $y^i$  to  $t^i$ .
- page 229, Exercise 29(a), line 4. Change “partitions” to “compositions”.
- page 235, Exercise 10(b), line 9. Change  $E(n)/n!$  to  $E(n)/n^n$ .
- page 237, Exercise 19. Change  $\binom{2n-k}{k}$  to  $\frac{2n}{2n-k} \binom{2n-k}{k}$ .
- page 237, line 2–. Change second  $a_{n-k+1}$  to  $a_{n-k+2}$ .
- page 245, line 8. The notation card  $t$  is not defined in the text, though it does appear on page 581.
- page 248, line 5–. Change “an” to “a”.
- page 249, Proposition 3.3.2, line 1. Change “condtions” to “conditions”.
- page 251, line 1–. Change  $\mathcal{I}$  to  $S$ .
- page 252, line 3. Insert “simple” before “matroids”.
- page 253, line 12–. Change  $t$  to  $v$  (three times).
- page 253, line 9–. In the definition of finitary distributive lattice, add the condition that  $L$  has finitely many elements of each rank. (Thus for instance the distributive lattice of all finite subsets of an infinite set is locally finite with  $\hat{0}$  but not finitary.)
- page 274, line 1. Change “equation (3.27)” to “the equation above (3.23)”

- page 277, line 12–. Change  $\dim(W \cup W')$  to  $\dim(W + W')$ .
- page 283, lines 8– to 6–. Delete the sentence “Let  $\Lambda_t = \{s \in L(\mathcal{A}) : s \leq t\}$ , the principal order ideal generated by  $t$ .”
- page 284, caption to Figure 3.21. Change  $\mathcal{A}^K$  to  $\mathcal{A}^u$ .
- page 284, line 5–. In order to make the title “Deletion-Restriction” of Proposition 3.11.5 clearer, one should define  $\mathcal{A}'$  to be the *deletion* of  $H_O$  from  $\mathcal{A}$  and  $\mathcal{A}''$  to be the *restriction* of  $\mathcal{A}$  to  $H_0$ .
- page 284, line 3–. Delete “real” at the end of the line.
- page 284, line 2–. Insert “over  $K$ ” after “arrangements”.
- page 285, line 9. Delete “ $\#\mathcal{B}_1 = \#\mathcal{B} - 1$  and”.
- page 285, line 11. Under the second  $\Sigma$ , change  $\mathcal{B}_1 \in \mathcal{A}''$  to  $\mathcal{B}_1 \subseteq \mathcal{A}''$ .
- page 288, proof of Proposition 3.11.9. Bruce Sagan points out that it would be simpler to say that every subspace in  $L(\mathcal{A})$  over  $\mathbb{Z}$  is defined by a row-reduced echelon matrix over  $\mathbb{Z}$ , so we have good reduction over any prime  $p$  larger than all the entries of this matrix.
- page 291, line 10–. Change “sketch that” to “reference for”.
- page 303, Theorem 3.15.8. To be completely accurate, one should assume that  $P \neq \emptyset$ .
- page 308, line 11. Insert “and equation (3.67)” after 3.15.7.
- page 308, line 9. Change “an” to “and”.
- page 311, line 11 (beginning  $n = 6$ ): change 0 to 2.
- page 312, line 12–. Change the second  $=$  to  $-$ .
- page 313, Figure 3.33. The poset  $P_5$  is drawn incorrectly.
- page 315, line 7. Change  $a_s - a_{s+1}$  to  $a_s - a_{s-1}$  (twice after the brace).
- page 317, after (3.80). The phrase “(since intervals of Eulerian posets are Eulerian)” is unnecessary since the formulas under consideration hold for any graded poset with  $\hat{0}$  and  $\hat{1}$ .

- page 319, line 12–. Change  $Q_1 * Q_2 * \cdots * Q_r$  to  $R_1 * R_2 * \cdots * R_r$  (since  $Q_i$  already has been given another meaning).
- page 319, line 11–. Change the first  $Q_i$  to  $R_i$ .
- page 332, line 1–. Change  $\gamma_{j-1}$  to  $\gamma_{j-1}^2$ .
- page 332, bottom of page. Add the following paragraph:  
The proof that  $(\gamma_j^*)^2 = 1$  is completely analogous. It also follows from the fact that  $\gamma_j^2 = 1$ , since we can assume without loss of generality that  $j = p - 1$  and then apply the automorphism of  $G$  that sends each  $\tau_k$  to  $\tau_{p-k}$ .
- page 334, line 4. Change  $z\delta$  to  $z\delta_{p-1}$ .
- page 336, line 11. Change  $\varphi: \widehat{KP} \rightarrow \widehat{KP}$  to  $\varphi: KP \rightarrow \widehat{KP}$ . page 337, line 3–. Change  $f(U)$  to  $f(y)$ .
- page 337, lines 14– to 11–. The sentence “For the algebraically minded . . . formal power series.” is not correct. We first need to consider non-commutative *polynomials* (not power series) and then pass to suitable completions.
- page 338, line 1–. This statement might not be so clear. Now  $e^{Dx}f(U)$  is an operator, operating on power series in  $U$ , where  $D$  acts as  $r\frac{d}{dU}$ . Let  $e^{Dx}f(U)$  act on  $g(U)$ . The Taylor series expansion gives

$$\begin{aligned} e^{Dx}f(U)g(U) &= f(U + rx)g(U + rx) \\ &= f(U + rx)e^{Dx}g(U), \end{aligned}$$

so  $e^{Dx}f(U) = f(U + rx)e^{Dx}$ .

- page 342, line 6. Change  $(1 - q)^2$  to  $(1 - q)$  (in the denominator).
- page 345, line 4. Change  $p_{s-1} + p_s$  to  $p_{j-1} + p_j$ .
- page 345, line 13. Change  $A$  to  **$A$**  (boldface).
- page 353, Exercise 3(d).
- page 354, Exercise 7(a), line 1. Change “the longest” to “a longest”.

- page 356, Exercise 15(g), line 3. Change 9655 to 11586.
- page 356, Exercise 16(b). Change the difficulty rating to [3] (or maybe [3+]). See M. Guay-Paquet, A. H. Morales, and E. Rowland, DMTCS proc. **AS** (FPSAC 2013 Paris) (2013), 253–264.
- page 358, Exercise 31(b,c,d). It should be assumed that  $L$  has rank at least three.
- page 360, Exercise 38. This exercise is incorrect for  $1 < k < m$ , where  $m$  is the maximum size of an antichain of  $P$ . A counterexample is given by  $k = 2$  and  $L = J(P)$ , where  $P = \mathbf{1} + (\mathbf{1} \oplus (\mathbf{1} + \mathbf{1}))$ . It is true that  $\#P_k = \#R_k$ , even for modular lattices. See Exercise 3.101(d).
- page 360, Exercise 41(c,d). It is assumed that  $L$  is distributive.
- page 360, Exercise 41(d), line 2. after  $t_i$  insert “,  $0 \leq i \leq m$ ”.
- page 363, Exercise 50, line 4. Change  $i \leq j \leq 0$  to  $i \geq j \geq 0$ .
- page 363, Exercise 51, line 2. Not an error, but to be consistent with the rest of the exercise it would be better to use  $n$  rather than  $i$  (twice).
- page 364, Exercise 55(b). The rating should be changed to [3–]. An exceptionally elegant proof was given by G. Stachowiak, *Order* **5** (1988), 257–259. Another elegant proof was given by B. Iriarte, [arXiv:1405.4880](https://arxiv.org/abs/1405.4880). Iriarte proves the more general result that if  $G$  is the comparability graph of a poset  $P$ , then the number of linear extensions of  $(G, \mathbf{o})$  is maximized when  $\mathbf{o}$  respects the order  $P$ .
- page 366, Exercise 62(g), line 2. Change  $m \geq 0$  to  $n \geq 0$  (under the summation sign).
- page 366, Exercise 62(g), line 2. Change  $U_{P_n}(x)$  to  $G_{P_n}(x)$ , where  $G_{P,\omega}(x)$  is defined in Section 3.15.2, and as usual we suppress the notation  $\omega$  in the case of natural labelings. Equivalently, for any finite labelled poset  $(P, \omega)$ ,

$$U_{P,\omega}(x) = \lim_{m \rightarrow \infty} U_{P,\omega,m}(x).$$

- page 374, Exercise 95. The stated identity is actually valid for the *dual* Möbius algebra, where one uses the join operation rather than the meet. The correct identity using the text definition of Möbius algebra is

$$\sum_{t \in L} \mu(t, \hat{1})t = \left( \sum_{u \geq z} \mu(u, \hat{1})u \right) \cdot \left( \sum_{v \vee z = \hat{1}} \mu(v, \hat{1})v \right).$$

- page 375, Exercise 96(d). This result was proved by Braden, Huh, Matherne, Proudfoot and Wang (difficulty rating [4]). For a survey, see T. Braden, J.P. Matherne, and N. Proudfoot, *Notices AMS*, to appear; <https://pages.uoregon.edu/njp/whatisNotices.pdf>.
- page 375, Exercise 98, line 3. Change  $\leq$  to  $\geq$ .
- page 376, Exercise 101(f). Difficulty level [2+].
- page 378, Exercise 114(a). The answer should be

$$(x-1)(x-2) \cdots (x-n).$$

- page 379, Exercise 115(c), line 1. Change “0,1” to 0. (There are  $\binom{n}{2}$  hyperplanes.)
- page 379, Exercise 116. The subscripts should not be in boldface.
- page 385, Exercise 131, lines 10–13. The definition of the partial ordering is incorrect. We should define  $\alpha \leq \beta$  if every block  $B \in \beta$  is a union of blocks  $A_{i_1}, \dots, A_{i_k}$  of  $\alpha$  (and that the restrictions of the labelings agree).
- page 387, Exercise 3.139. The right-hand side of the displayed formula should be  $2x(1+x)^{n-2}/(1-x)^n$ .
- page 387, Exercise 141, line 3. Change  $\sum_{i \geq 1}$  to  $\sum_{i \geq 0}$ .
- page 387, line 7–. It should be said that  $Q_0$  is defined in the previous exercise.
- page 388, Exercise 145, line 3. Change 3.15.10 to 3.15.12.
- page 388, Exercise 149, line 4. Change  $1 \leq i, k \leq r$  to  $1 \leq i, j \leq r$ .

- page 390, Exercise 158(a), line 4. Change **a.** at beginning of line to **i.**
- page 390, Exercise 158(a), line 5. Change **b.** at beginning of line to **ii.**
- page 390, Exercise 158(c). Delete one of the periods at the end of the line.
- page 390, Exercise 158(d), line 2. Change “thats” to “that”.
- page 390, Exercise 158(e), line 1. Change  $k(kt)!^{t-2}$  to  $k(kt)^{t-2}$ .
- page 403, Exercise 189. NOTE. The result is also true for  $d$  odd, but the proof is quite a bit easier.
- page 404, Exercise 198. It should be mentioned that butterfly posets are also called “ladder posets,” since there is an index entry for “poset, ladder.”
- page 405, line 8–. The notation  $f_{00}$  is not defined until the next sentence.
- page 406, line 1–. Change  $\text{des}(w)$  to  $\text{asc}(w)$ .
- page 407, line 1. Change  $\text{des}(w)$  to  $\text{asc}(w)$ , and change “descents” to “ascents”.
- page 407, line 3. Change  $z \sum_{n \geq 1}$  to  $\sum_{n \geq 1}$  (twice).
- page 407, line 5. Change  $(x - 1)^{n-1}$  to  $(z - 1)^{n-1}$ .
- page 407, Exercise 205(b), line 2. Change 44605 to 44606 (private communication from Patrick Byrnes, 21 February 2012). Byrnes originally assumed that a vertex  $v$  could be covered by at most one singleton (element covering only  $v$ ), but there is exactly one example up to rank 9 where this property need not hold. Byrnes also computes that there are 29,199,636 1-differential posets up to rank 10.
- page 408, Exercise 211, line 3. Change the numerator  $r^2 + (r + 1)q - q^2$  to  $r^2 + r(r + 1)q - rq^2$ .



- page 408, Exercise 215(c). Change the difficulty rating [5] to [3]. See A. Miller, *Order* **30** (2013), 657–662, [arxiv.org/abs/1202.3006](https://arxiv.org/abs/1202.3006), and C. Gaetz and P. Venkataramana, *Order* **37** (2020), 279–286; [arxiv.org/abs/1806.03509](https://arxiv.org/abs/1806.03509).
- page 411, line 8–. Change **f.** to **g.**.
- page 411, line 8–. Change (e) to (f).
- page 412, Exercise 16, line 2. Update the reference to *J. Combin. Theory Ser. A* **120** (2013), 1305–1327.
- page 413, Exercise 26, line 2. We can’t replace  $\mathbb{N}^*$  with  $\mathbb{Z}$  since  $\mathbb{Z}$  doesn’t have a  $\hat{1}$ .
- page 415, Exercise 36(b). This example is incorrect. For instance,  $\{1, 2, 3\}$  and  $\{1, 2, 4, 5\}$  have no meet.
- page 416, Exercise 41(a), line 3. Change “dominance” to “lexicographic”.
- page 416, Exercise 47(b). Bruce Sagan points out that this exercise can be given a more intuitive, noninductive proof.
- page 418, line 5. To verify this identity, we must use the fact that  $2 \leq r \leq k$ . (We are assuming  $r > 1$ , and if  $r > k$  then the meet of elements covered by  $t$  has rank  $k - r < 0$ .)
- page 426, Exercise 87(d), line 1. In addition to Example 3.9.6, we are also using the fact (immediate from the definition of meet-distributive) that if  $[s, t]$  is an interval of a meet-distributive lattice for which  $s$  is the meet of elements in  $[s, t]$  covered by  $t$ , then  $[s, t]$  is boolean, together with Corollary 3.9.5.
- page 426, Exercise 87(d). One should define a chain  $t_0 < t_1 < \cdots < t_k$  to be *boolean* if every interval  $[t_{i-1}, t_i]$  is a boolean algebra. Also define  $\ell$  to be the length of the interval  $[t_0, t_k]$ .
- page 426, Exercise 88. Another solution is the following. We have

$$\sum_{s \leq t} \mu(s, t) = \sum_s \sum_{t \geq s} \mu(s, t).$$

By the definition (3.15) of  $\mu$ , this latter sum over  $t$  is 0 if  $t \neq \hat{1}$  and is 1 if  $t = \hat{1}$ , and the proof follows. Note that this proof only requires  $P$  to have a  $\hat{1}$ . More generally, we leave the reader to show that for *any* finite poset  $P$ ,  $\sum_{s \leq t} \mu(s, t) = 1 + \mu_{\hat{P}}(\hat{0}, \hat{1})$ , where  $\hat{P}$  denotes  $P$  with a  $\hat{0}$  and  $\hat{1}$  adjoined.

- page 427, Exercise 96(a), line 7. Change  $[F(s \wedge t), s]$  to  $[F(s \wedge t, s)]$ .
- page 429, line 9. Change “that” to “then”.
- page 429, line 12. The sum is not empty, but all terms  $u$  satisfy  $u \notin B$ . Hence  $f(u) = 0$ .
- page 429, line 13–. Change  $T^*$  to  $t^*$ .
- page 430, Exercise 102(a). Considerable further progress (but not a complete solution) has been made. See the Wikipedia article “Union-closed sets conjecture.”
- page 431, line 5. Change  $v \geq u$  to  $v \leq u$ .
- page 431, line 2–. Change **23** to **33**.
- page 434, line 9–. Change  $p > 2$  to  $p > 2n - 1$ .
- page 437, line 11. Not the best notation, since  $x \parallel y$  also means that  $x$  and  $y$  are incomparable in a poset.
- page 439, Exercise 130, line 1. insert  $-\{\hat{0}\}$  after  $Q_i$ .
- page 440, Exercise 135, line 1. Change 3.12 to 3.13.
- page 445, Exercise 151, line 3. Change the second  $+1$  (subscript) to  $+2$ .
- page 449, Exercise 163(a), line 3. Change  $2e_P(p-1) - (p-1)e(p)$  to  $(2e_P(p-1) - (p-1)e(P))/p!$ .
- page 458, Exercise 185. Part **f.** should be **k.**, and **g.** should be **l.**
- page 459, line 5–. Should be  $\beta_P(4, 5, 6) = -1$ .
- page 465, line 7–. Change “degree  $j$ ” to “degree  $j - 1$ ”.

- page 468, Proposition 4.2.2, line 5. Change the period after  $E_f$  to a comma.
- page 469, line 6–. Change  $\sum_{n \geq 0} f(n)$  to  $\sum_{n \geq 0} f(n)x^n$ .
- page 511, line 18–. Change  $(v, b_i)$  to  $(v, i)$ ,  $1 \leq i \leq k$ .
- page 522, Figure 4.29. The vertex labels should be 0 and 1, not 00 and 10.
- page 528, Exercise 4.2, line 3. Change “over” to “in”.
- page 530, Exercise 4.12. Change  $0.00010203050813213455 \dots$  to  $0.0001010203050813213455 \dots$ , and change 1, 2, 3 to 1, 1, 2, 3.
- page 530, Exercise 4.14, line 2. Insert “relatively prime” after “choose  $a, b$ ”.
- page 531, Exercise 22, line 1. Change  $\alpha \in \mathbb{C}$  to  $0 \neq \alpha \in \mathbb{C}$ .
- page 532, Exercise 25(e), line 3. Change  $N_1(m)$  to  $N_1(2^m - 1)$ .
- page 532, Exercise 25(e), line 5. Change  $N_1(m)$  to  $N_1(q^m - 1)$  and  $N_{-1}(m)$  to  $N_{-1}(q^m - 1)$ .
- page 532, Exercise 25(e), line 8. Change  $N_1(m)$  to  $N(2^m - 1)$ .
- page 532, Exercise 25(e), line 10. Change  $N_1(m)$  to  $N_1(2^m - 1)$ .
- page 533, line 2. Change  $N_1(m)$  to  $N_1(2^m - 1)$ .
- page 533, line 5. Change  $N_1(m)$  to  $N_1(2^m - 1)$ .
- page 533, line 7. Change  $N_1(m)$  to  $N_1(3^m - 1)$ .
- page 533, line 8. Change  $N_2(m)$  to  $N_2(3^m - 1)$ .
- page 533, line 10. Change  $N_1(m)$  to  $N_1(3^m - 1)$ .
- page 533, line 11. Change  $N_2(m)$  to  $N_2(3^m - 1)$ .
- page 539, Exercise 47, line 1–. Change 1.1.8.6 to 1.8.6.
- page 542, line 4. Change  $n \times n$  to  $(n + 1) \times (n + 1)$ .

- page 542, line 10. Delete ) after  $u_6$ .
- page 544, Exercise 4.66(b), line 2. Change “each  $i$ -face” to “the affine span of each  $i$ -face”.
- page 546, Exercise 4.75(d). Change the difficulty rating to [3–]. A combinatorial proof (though not a simple bijection) was found by Jacob Scott, private communication dated September 2, 2012.
- page 548, Exercise 2(a), line 6. The assertion “Clearly, we can write  $f(x) = P(x)/Q(x)$  for some relatively prime integer polynomials  $P$  and  $Q$ .” is not so clear. One must show that if  $F(x) \in \mathbb{Q}[[x]]$  and  $F(x) = R(x)/S(x)$  where  $R, S \in \mathbb{C}[x]$ , then one can write  $F(x) = P(x)/Q(x)$  where  $P, Q \in \mathbb{Q}[x]$ . This statement does have a fairly simple proof, which we leave as an exercise.
- page 549, line 1–. Change  $x^{m^2}$  to  $x^{n^2}$ .
- page 552, Exercise 14. The smallest known pair  $(a, b)$  seems to be

$$\begin{aligned} a &= 106276436867 = 31 \cdot 3128272157 \\ b &= 35256392432 = 2^4 \cdot 2203524527 \end{aligned}$$

due to M. Vsemirnov, *J. Integer Seq.* **7** (2004), article 04.3.7.

- page 557, line 4–. Change  $x^\ell$  to  $x^l$ .
- page 558, line 1. Delete comma after *Wochenschrift*.
- page 558, Exercise 43. The sequence  $t(3), t(4), \dots$  is known as *Alcuin’s sequence*, after Alcuin of York (730s or 740s – 19 May 804). For a survey see D. J. Bindner and M. Erickson, *Amer. Math. Monthly* **119** (2012), 115–121.
- page 560, Exercise 46(b), line 1. Delete the first “many”.
- page 560, Exercise 46(b), line 2. Change  $f(n+1), f(n+2), \dots, f(n+d)$  to “ $(f(n+1), f(n+2), \dots, f(n+d))$  for fixed  $d$ ”.
- page 564, Exercise 58(f), line 1. Change “order polynomials” to “Ehrhart polynomials”.

- page 567, Exercise 75(c), line 1. Delete “directed”.
- page 569, Exercise 80, displayed equation, line 1. Change  $v'_iy$  to  $v_iy$ .
- page 570, Exercise 82(e,f). For the paper of J. Schneider, see *Electronic J. Combinatorics* **21** (2014), P1.43; [arXiv:1206.6174](#).
- page 571, line 3. Change “chose” to “chosen”.
- page 578. The following item is missing: First edition—Supplementary Exercise 1.7 = Second edition—Exercise 1.114(b).
- page 579. The following three items are missing: First edition—Supplementary Exercise 3.19 = Second edition—Exercise 3.44; First edition—Supplementary Exercise 3.10 = Second edition—Exercise 3.63; and First edition—Supplementary Exercise 3.17 = Second edition—Exercise 3.189.
- page 620, Stembridge entry, line 6. Change  $q = 1$  to  $q = -1$ .