6 Homework Solutions

18.335 - Fall 2004

6.1 Let A be skew Hermitian, i.e. $A^* = -A$. Show that $(I - A)^{-1}(I + A)$ is unitary.

See solutions for the first Homework, problem 2.

6.2 Trefethen 25.1

(a) Let λ be an eigenvalue of A. Therefore $B = A - \lambda I$ is singular and hence

$$\text{rank}(A - \lambda I) \leq m - 1$$

The $m - 1 \times m$ submatrix $B_{2:m,1:m}$ is upper triangular whose diagonal entries are non-zero by our assumptions on A. Hence $B_{2:m,1:m}$ has $m - 1$ linearly independent columns which implies

$$\text{rank}(B_{2:m,1:m}) = m - 1$$

Therefore we must also have $\text{rank}(A - \lambda I) = m - 1$, and hence the null space of B is spanned by one vector, a unique eigenvector of A correspoding to λ. Since A is Hermitian, which requires m linearly independent eigenvectors, all λ must be distinct.

(b) $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$