Professor Haynes R. Miller, MacVicar Fellow

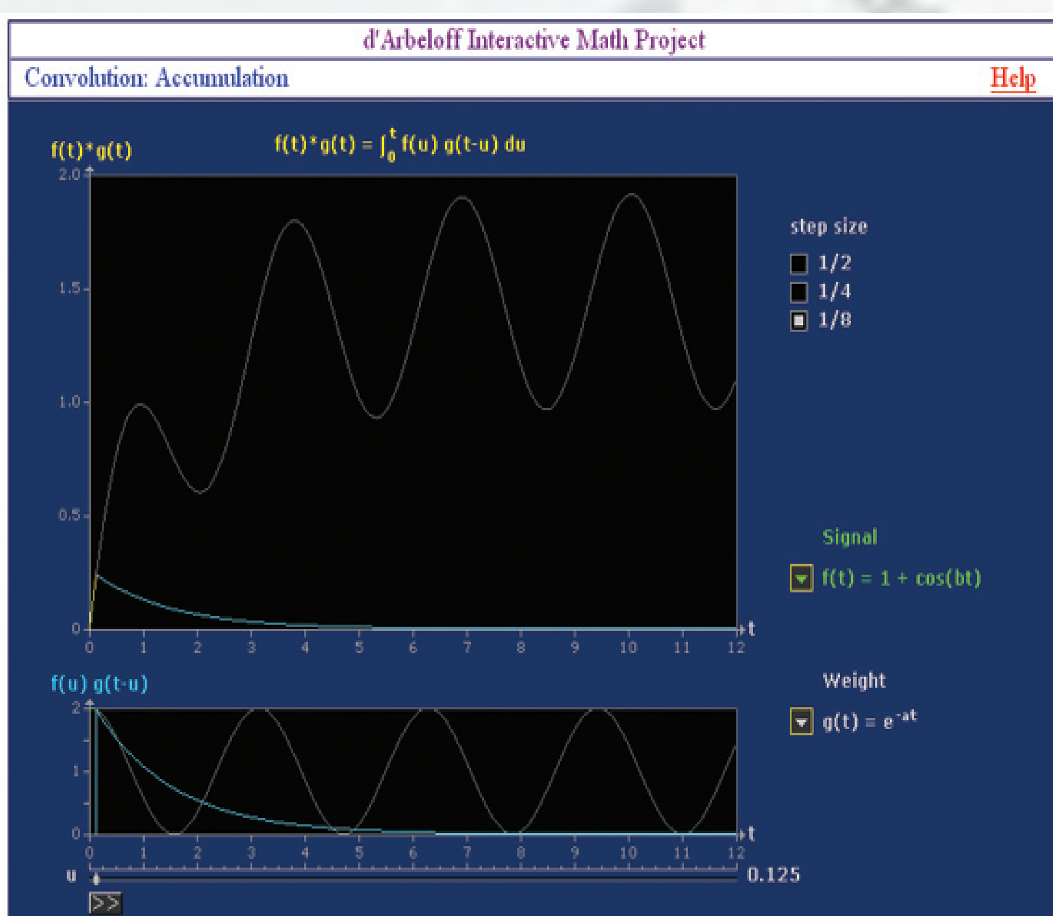
Department of Mathematics

"An expert looks at an equation, and it tells a story. Each symbol tells part of the story. But students often just see a mess."

"To understand the drama of an equation, one needs to know how the characters behave in different situations. The expert can substitute values for the symbols and visualize the effect. I want students to be able to move around in the variable-space; to see -- and I mean SEE -- how things change when the parameters change. I want them to begin to hear the story an equation tells, and to move away from just pushing the formulae around." - Professor Miller

Directed use of Mathlets - Explore Mathlets at http://www-math.mit.edu/daimp

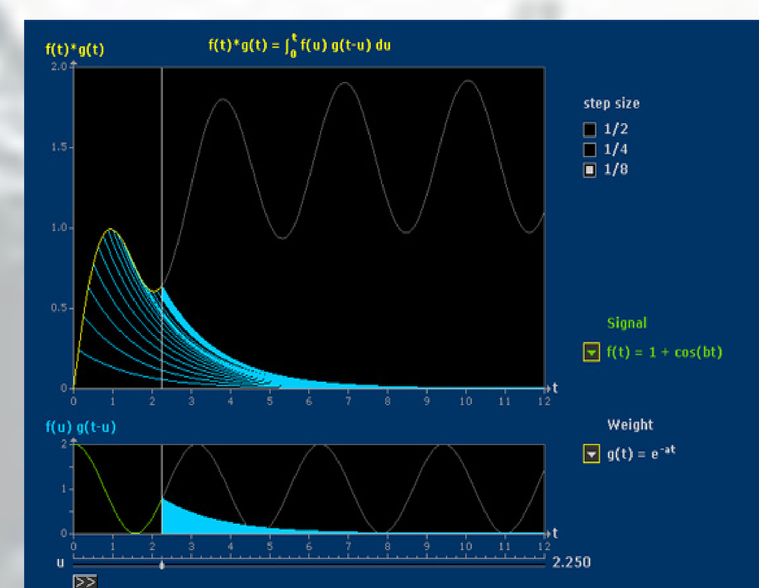
Mathlets (computer manipulatives) are used to help students engage with and more thoroughly understand specific mathematical concepts. Through guided use of the mathlets, students developed a better sense for the roles that specific parameters play in the physical manifestations of individual equations.



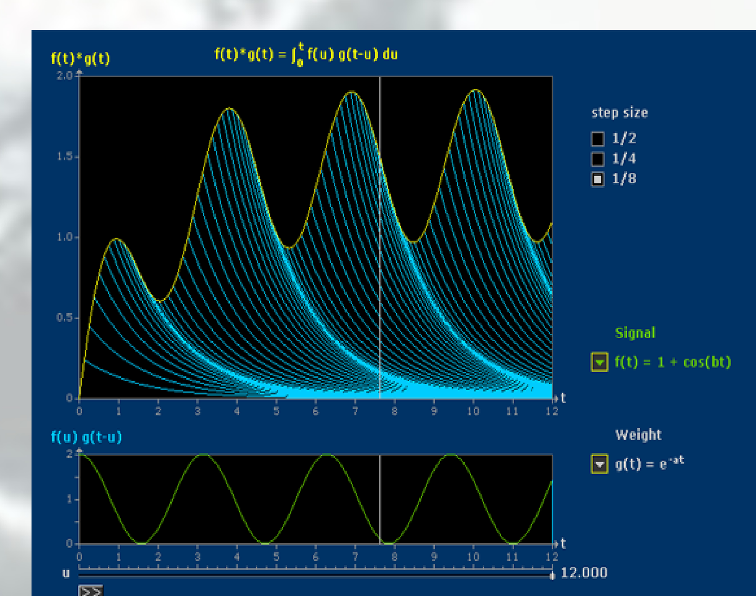
After a time interval of 1/8 = .125 , (.125) x 2 = .25 tons of phosphate has entered the lake. It decays exponentially (carried away by a stream, or decomposed), as shown.

Convolution: Accumulation Mathlet

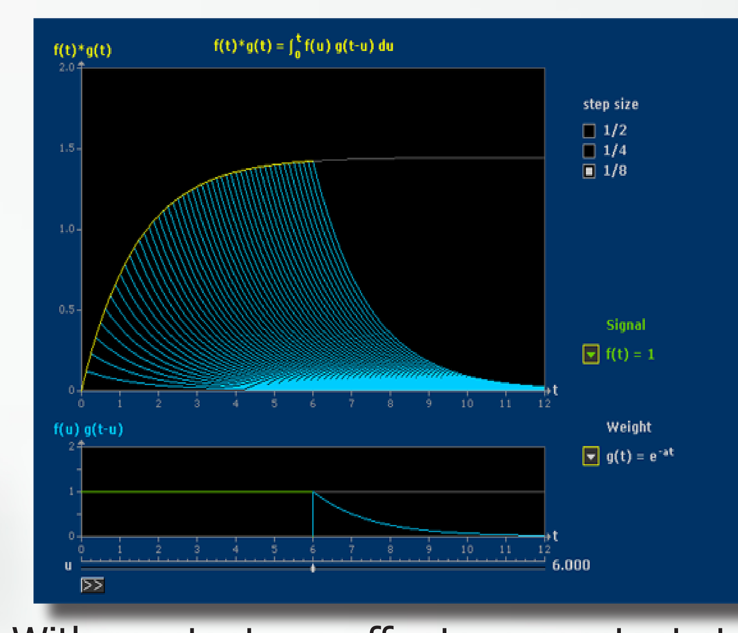
Convolution can be understood as modeling farm run-off into a lake. The rate of phosphate, say, in tons per year, is seasonal and is shown in the bottom graph. The lake starts clean and the runoff rate at a maximum.



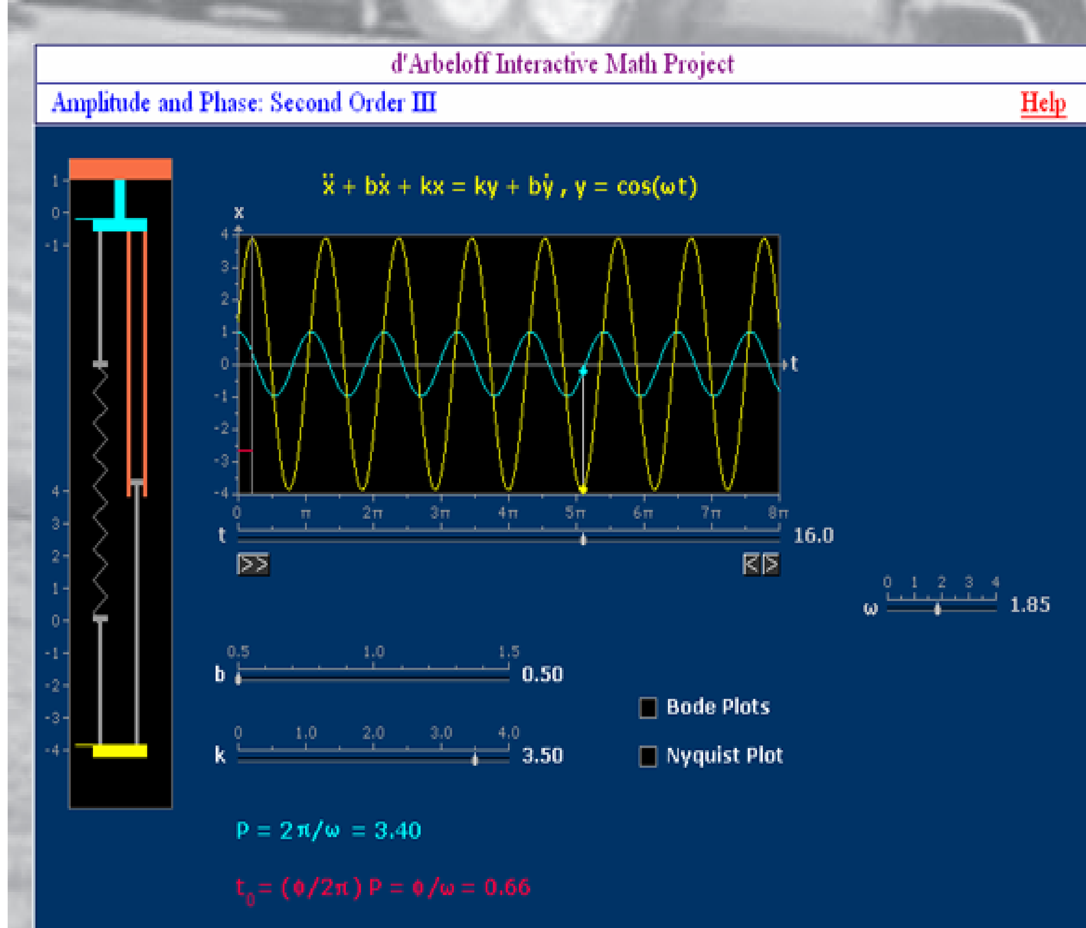
Time has passed; the early contributions have decayed, and more have been added. You can see the current addition, and its future, in solid blue. The sum of these decayed contributions is given by the convolution integral.



After a few years, the transient has died off and a steady sinusoidal pattern is established.



With constant run-off rate, a constant steady state is reached.
Choosing t = 6 shows the effect on the phosphate level in the lake if the pollution is stopped at that moment.

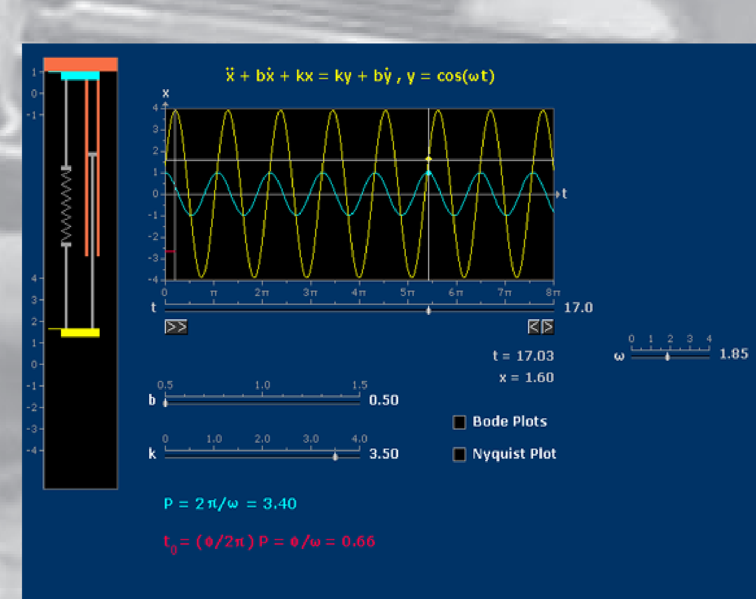


Near resonance! The displacement of the mass far exceeds that of the driving plunger.

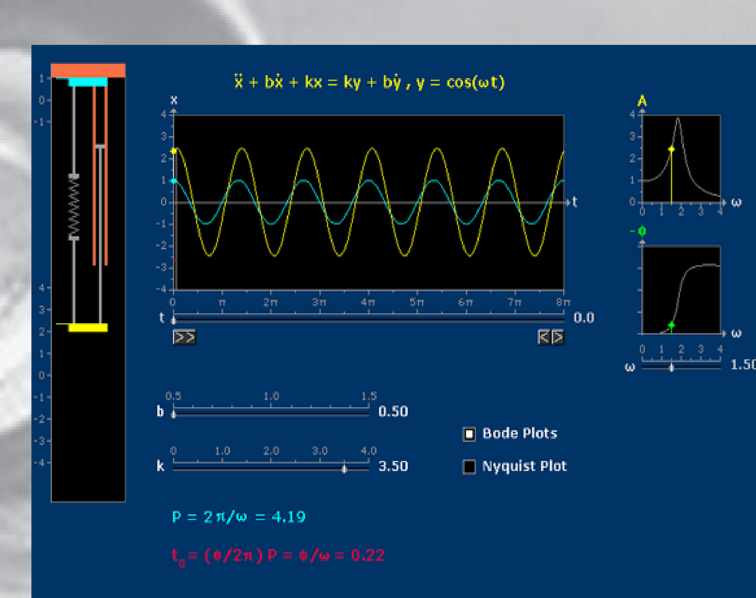
Amplitude & Phase: Second Order III

By spring, Cambridge streets are well approximated by washboard sinusoids. A simplified car suspension is represented by a system with a spring and dashpot in parallel. This is shown on the mathlet, but upside down, and is modeled by the displayed equation. Sliders control the strength of the spring and of the dashpot, and the system can be animated.

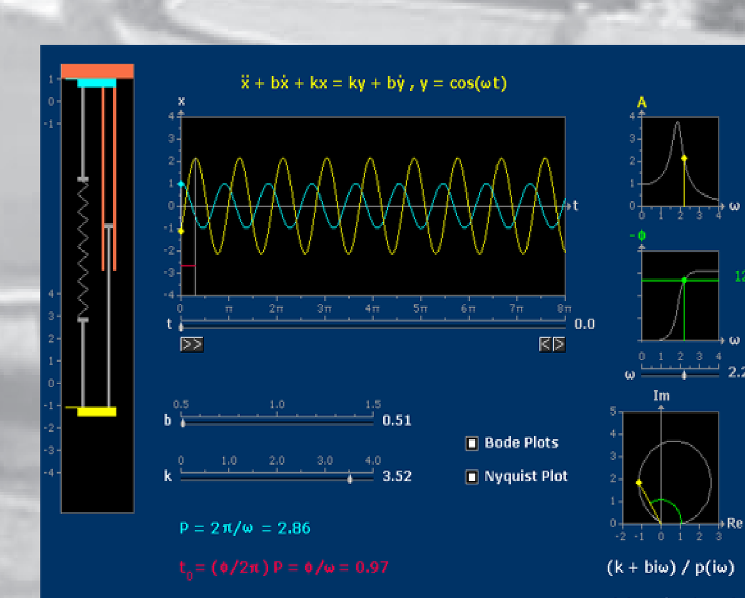
We are all familiar with the near resonance shown at left!



One second later, the spring has compressed beyond its relaxed position. Crosshairs allow the student to make measurements.



With the Bode plot toggled, the student can see the resonant peak near omega = 2. For smaller omega, the system response is in phase with the input signal.



As the input signal frequency is pushed past resonance, the phase lag increases sharply. Amplitude and phase lag are magnitude and negated argument of the "complex gain," pictured as a complex number at lower right.

A Creative Group Came Together to Bring Mathlets to Life

Pictured L-R: Daniel Kleitman, Andrew McKinney, Ashot Heyrapetyan, Hu Hohn, David Jerison, Haynes Miller, Jean-Michel Claus, Gilbert Strang (not pictured: Deborah Upton)

What Do Sudents Say About Mathlets?

KEEP DOING THE VISUALS!! I always remember on tests pictures from visuals, plus it adds a dimension to learning. —— I found my biggest problem was that I didn't really understand what the graphs meant from class/reading. Thus, playing with the graph and having to use them really forced me to understand them, and I would DEFINITELY recommend continuing to use them. It's funny that I didn't even realize how much they helped until you are trying to think about it later and you remember the graph. —— Sometimes just calculating something is not very effective in helping to understand it. I think the visuals were very good, in general, in helping me understand concepts and think about concepts rather than just follow a standard method of solving a problem. —— I never really understood the concept of convolution (and the decay) until I saw the visual. —— I have always thought that the visuals were the most fun part of the problem set. It is always satisfying to see that the "experimental" value is not too far off from the theoretical value. —— I enjoyed the way the VibrationAmplitudePhase portrayed the external force and how it worked with varying frequencies. —— The VibAmpPhase is really cute. My friend said he wanted to try to make it his personal screen saver. ——I used it to study for a test and I got a 97!