

Part I (20 points)

Lecture 32. Thurs. Jan. 25. Divergence theorem cont'd. Applications, interpretations.

Read: Notes V15 sec.1 for div in ∇ notation - skip Stokes' theorem refs.; Notes v15 sec. 2 to middle p.3

Work: 6C - 10, 11; 6H - 1, 3a

Lecture 33. Fri. Jan.26 Line integrals in 3D; conservative fields, potential functions.

Read: Notes V11, V12 Work: 6D - 1b, 2, 4, 5; 6E - 3(ii) (b: method 1), 5 (method 1)

Lecture 34. Mon. Jan.29 Stokes' Theorem

Read: Notes V13 Work: 6E - 6bc (method 2); 6F - 1b, 2, 5

Lecture 35. Tues. Jan. 30 Stokes' Theorem continued.

Read: Notes V14 Work: 6G - 1

Lecture 36. Wed. Jan. 31. Applications of vector integral calculus. Review.

Problem Set 8 due. No late papers: solutions posted during class.

Read: Notes V15

Thurs. Feb. 1 **18.02A Exam** covering 18.02A 2nd half (all material covered during IAP)
WALKER 3rd Floor, 12-2pm

Part II (30 points)

Problem 1. (Thursday. 6pts) Confirm the Divergence Theorem in the region $0 \leq z \leq 10 - \sqrt{x^2 + y^2}$ for the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ by calculating each side of the formula directly.

Problem 2. (Fri. 6pts: 2+2+2)

a) Compute (in terms of the constants a, b) the work done by the vector field $\mathbf{F} = (a \sin z + bxy^2)\mathbf{i} + x^2y\mathbf{j} + (x \cos z - z^3)\mathbf{k}$ along the portion of the helix $x = \cos t$, $y = \sin t$, $z = t$ from $(1,0,0)$ to $(1,0, 2\pi)$.

b) Compute curl \mathbf{F} . For which values of a and b is the vector field \mathbf{F} conservative?

c) Let a and b be the values you found above. Find a potential function for \mathbf{F} using a systematic method, and verify the answer you found in part a) using the fundamental theorem of calculus.

Problem 3. (Mon. 10pts: 5+5)

Let R be the region under the surface $z = 1 - x^2 - y^2$ in the first quadrant. The boundary of R has four surfaces, S_{xy} , S_{xz} , S_{yz} and S_{top} , where the first three lie on the xy -, xz - and yz -planes, and the fourth is the curved top surface. These four surfaces meet in six curves, $C_x, C_y, C_z, C_{xy}, C_{xz}$ and C_{yz} , where the first three lie on the respective axes, and the latter three on the respective planes. Let $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + (z - x - y)\mathbf{k}$.

a) Orient each of the above four surfaces outward from R . Compute $\int \int_S \nabla \times \mathbf{F} \cdot d\mathbf{S}$ for each of these surfaces.

b) For each of these four oriented surfaces, calculate the line integral around its boundary, oriented according to the right-hand rule. Save effort where you can. Verify Stokes' Theorem in each of these four cases. Explain why the sum of the four surface integrals is zero, using a simple observation about the equivalent line integrals.

Problem 4. (Mon. 4pts: 2+2)

Suppose that in 3-space, $\mathbf{F} = \nabla \times \mathbf{G}$, where the components of \mathbf{G} have continuous second partial derivatives.

Prove in two different ways that if S is a closed, positively-oriented surface, $\int \int_S \mathbf{F} \cdot d\mathbf{S} = 0$:

a) use the Divergence Theorem,

b) divide S into two parts with a closed curve C and apply Stokes' Theorem.

Problem 5. (Mon. 4pts: 2+2)

Let $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Show in two different ways that there cannot be a field \mathbf{G} such that $\mathbf{F} = \nabla \times \mathbf{G}$:

a) Let S be a sphere of radius a centered at the origin, and C be a simple closed curve on S . Using Stokes' Theorem, interpret the value of $\oint_C \mathbf{G} \cdot d\mathbf{r}$ geometrically, and show this leads to a contradiction.

b) Find another argument (look over the exercises in 6H for ideas).