18.02A Problem Set 5 – IAP 2007 due Friday Jan.13, 11:45 in 2-106

Part I (20 points)

Lecture 40. Mon. Jan. 9 Vector fields and line integrals in the plane. Work.
Read: Notes V2, Simmons 21.1 Work: 4A-3bd, 4; 4B-1bef, 2, 3
Lecture 41. Tues. Jan. 10 Gradient fields, conservative fields, path-independence; Fundamental Theorem of Calculus for line integrals.
Read: 21.2 to p.762 Work: 4C-1,2,3
Lecture 42. Wed. Jan. 11. Criterion for conservative fields; potential functions.
Read: Notes V2 Work: 4C-5a, 6ab (do all 3 by Method 1 and Method 2)
Lecture 43. Thurs. Jan.12 Green's Theorem (tangential form)
Read: 21.3 to middle p. 768 Work: 4D-1bc, 2, 3, 5.
Lecture 44. Friday Jan.13 Flux; Green's theorem in normal form
Read: Notes V3, V4

Part II (30 points)

Directions. Try each problem alone for 15 minutes. If you subsequently collaborate, solutions must be written up independently. It is illegal to consult problem sets from previous semesters. With each problem is the day it can be done.

Problem 1. (Tues. 6pts: 3+3) Consider $\mathbf{F} = (4xy + y^2 + 2)\mathbf{i} + (2x^2 + 2xy + y^2)\mathbf{j}$. Find a potential for $\mathbf{F}$ in two ways: a) the algebraic method, and b) the integration method.

Problem 2. (Tues. 6pts: 2+2+2) Consider $\mathbf{F} = xy\mathbf{i} + xy\mathbf{j}$.
a) Show that $\mathbf{F}$ is not a gradient field by taking the appropriate derivatives.
b) Try to find a potential function for $\mathbf{F}$ by the integration method. What goes wrong?
c) Same question for the algebraic method.

Problem 3. (Wed. 6 pts: 3+3)
a) Calculate the curl of $\mathbf{F} = r^n(x\mathbf{i} + y\mathbf{j})$.
b) For each n for which curl $\mathbf{F} = 0$, find a potential g such that $\mathbf{F} = \nabla g$. (Hint: seek a potential $g = g(r)$. Watch out for a certain negative n value for which the formula is different.)

Problem 4. (Wed. 6 pts: 2+4) Consider $\mathbf{F} = \nabla(x^2y + xy^2)$. Let C be the semicircle having its midpoint at the origin and running from (-1,1) to (1,1).
a) Write the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ in the $\int_c M\,dx + N\,dy$ form.
b) Evaluate the integral in two different but easy ways: using i) the Fundamental Theorem, and ii) path-independence. Show the calculations in each case.

Problem 5. (Thurs. 6 pts: 4+2)
a) For what simple closed (positively oriented) curve C in the plane does the line integral $\oint_C (x^2y + y^3 - y)dx + (3x + 2y^2x + e^y)dy$ have the largest positive value. (Hint: use Green's Theorem).
b) What is this maximum value?