

18.02A Problem Set 4 – Fall 2006 due Thursday Dec.7/06, 12:45 in 2-106

Part I (15 points)

Lecture 33. Thurs. Nov. 30 16 Chain Rule.

Read: 19.6 (skip Example 4) Work: 2E - 1b, 2c, 3b, 5a, 7, 8a

Lecture 34. Friday. Dec. 1 Chain Rule for non-independent variables.

Read: Notes N.1-N.3 Work: 2J - 1a, 2i, ii (for (a) only), 3a, 4a, 5a, 6, 7

Lecture 35. Tues. Dec.5 Double and iterated integrals in rectangular coordinates

Read: 20.1, 20.2 Work: 3A - 1ad, 2b, 3b, 4c, 5a

(The following exercises are not to be handed in, but are suggested as practice problems, to help prepare for the midterm.)

Lecture 36. Thurs. Dec.7 Double integrals in polar coordinates; physical applications

Read: 20.4 Work: 3B - 1ac, 2cd, 3bc

Read: 20.3 Work: 3C - 1ac, 2, 4, 5

Lecture 37. Fri. Dec.8 Change of variables in double integrals

Read: Notes CV Work: 3D - 1, 2, 3, 4

Lecture 38. Tues. Dec.12 Continuation and review of double integrals

MIDTERM Thursday Dec.21 9:00 - 11:00 AM Johnson A.C.

Roughly 1/4 on material covered on last test, 3/4 on that covered since.

Grading

You will get a grade for 18.02A first half, and a separate grade for 18.02A second half. The two will be averaged for your final grade. If you finish during the spring semester, this grade (if A, B or C) will appear on your transcript. If you finish during IAP, it will not; only P (pass) will be recorded. If you get a D or F, nothing will appear on the transcript regardless of when you finish, provided you are still a freshman.

Registration for Second Half of 18.02A

Register on-line, as for other courses; select either IAP or Spring. You should have passed the first half. Students who receive F in the first half cannot take the second half; those who receive D and wish to continue will be considered on a case-by-case basis.

For IAP 18.02A, there are two recitation times: 10-11:30 or 2-3:30 on Tues. and Thurs. (Directions for choosing will be given).

Lectures will be 12-1 everyday, starting Monday Jan. 8; there will be one exam in the middle (Friday Jan.19), and a two hour final exam on the last day of IAP (Fri. Feb. 2) covering the second half of 18.02A.

Part II (20 points)

Problem 1. (Thurs. 3 pts)

When $f(x, y) = e^x \cos y$ and $x = t^2$, $y = 1 - t^3$, find $\frac{\partial^2 f}{\partial t^2}$.

Problem 2. (Thurs. 4pts)

Let $u = f(x, t)$ and $r = x - ct$, $s = x + ct$. Show that

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial r \partial s}$$

Problem 3. (Friday. 5pts: 2+2+1)

If (r, θ) are standard polar coordinates and (x, y) standard Cartesian coordinates, Consider the function $w = (r^2 - x^2)^{1/2}$.

a) Calculate $(\frac{\partial w}{\partial r})_\theta$ by:

i) writing w in terms of r and θ ,

ii) writing w in terms of r and x and using the Chain Rule. Show your answer is the same as from (i) when expressed entirely in terms of r and θ .

b) Study Example 6 in Notes N.3 then, without referring to the notes, write down the derivation of the final formula given there. Then calculate $(\frac{\partial w}{\partial r})_\theta$ by substituting into this formula.

c) Finally, obtain the answer by intuitive geometrical reasoning, noting the relation between w and y .

Problem 4. (Tues. 4pts)

A rectangular prism has a rectangular base, four vertical sides and a top corresponding to a portion of the plane $z = ax + by + c$. The prism lies entirely above the $x - y$ plane, and its four vertical edges can have different lengths. Show by double integration that

$$\text{Volume of prism} = (\text{area of base}) \times (\text{average of the lengths of the vertical edges})$$

Problem 5. (Tues. 20pts)

Change the order of integration to calculate

$$\int_0^1 dx \int_x^{2-x} \frac{x}{y} dy .$$

Hint: be sure to sketch the region of integration before proceeding.