18.02A Problem Set 3 – Fall 2006 due Thursday Nov.30/06, 12:45 in 2-106

Part I (20 points)

Lecture 29. Thurs. Nov. 16 Gradient; directional derivative.
Read: 19.5 to p. 683 Work: 2D - 1 acd, 2a, 4, 7, 9bcde
Lecture 30. Friday. Nov. 17 Gradients in 3D; tangent planes.
Read: rest of 19.5, Notes P Work: 2D - 1b, 2b, 3a, 5bc, 8; 2K - 3a, 5.
Lecture 31. Tues. Nov. 21 Max-min problems. Least squares approximation,
Read: 19.7 to bottom p.693; Notes LS Work: 2F - 1b, 5, 2G - 1c, 4.
Lecture 32. Tues. Nov. 28 Second derivative test. Lagrange multipliers.
Read: 19.7, p. 694-5 ; 19.8 Work: 2H - 1ad, 2I - 1a, 4a.

Part II (20 points)

Problem 1. (Thurs. 4pts: 2+2)
A hiker climbs a mountain whose height is given by $z = 1000 - 2x^2 - 3y^2$.
a) When the hiker is at the point $(1, 1, 995)$, in what direction should she move in order to ascend as rapidly as possible?
b) If she continues to move on a path of steepest ascent, show that the projection of this path on the xy-plane is $y = x^{3/2}$.

Problem 2. (Friday. 4 pts: 2+2)
Consider the vector field $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$. Show that $\mathbf{F}$ is not the gradient of any scalar function $f(x, y)$ by:
a) arguing geometrically on the basis of the definition of the gradient, and the form of **F**.
b) (Tues. after Thanksgiving) using properties of the second derivative.

Problem 3. (Tues. 6pts: 2+2+2)
Place a unit cube in the corner of the first octant, with edges along the axes. Each of its six faces contains a diagonal line; for this problem, use the front-face diagonal containing (1, 0, 1) and the right-side-face diagonal containing (0, 1, 0). These two lines are skew; the problem is to find the length and position of the shortest line segment joining them. This can be done by vector methods (cf. 1E-7), but here we shall treat it as a minimization problem in two variables.
a) Draw a picture and write parametric equations for the two lines containing these two diagonals, using in each case the given point as the base point on the line. To avoid confusion, use two different variables, t and u, as the parameters for the two lines.
b) Let $w(t, u)$ be the square of the length of a line segment AB joining a point A on the front diagonal with a point B on the side diagonal. Find the unique critical point (t_0, u_0) of the function $w(t, u)$, and from this determine the corresponding position and length of the minimal line segment A_0B_0. Draw it on your picture.
c) (Tues. after Thanksgiving) Using the second derivative test, verify that the length of line segment A_0B_0 represents a minimum value for the function $w(t, u)$, i.e. that it is neither a maximum nor a saddle.

Problem 4. (Tues. after Thanksgiving: 6pts)
Use the Lagrange Multiplier method to find the maximum and minimum values of $x^2 - 2xy + 7y^2$ on the ellipse $x^2 + 4y^2 = 1$. (First, clearly write down the equations you need to solve - which is worth half the credit - then solve them. It is easy to make a mistake in the algebra, so as a check, the answers should come out to be rational numbers.)