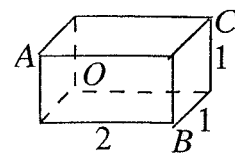


18.02A Practice Exam 1



Problem 1 (10) The picture shows a rectangular box, having length 2 and height 1 and width 1. Find the cosine of angle BOC.

Problem 2 (10) At what point (x, y, z) does the line given parametrically by

$$x = 1 + 2t, \quad y = 1 - t, \quad z = 2t$$

intersect the plane through the point $(1, -1, 3)$ and perpendicular to the vector $\mathbf{i} - \mathbf{j} - \mathbf{k}$?

Problem 3 (20) $\mathbf{a} = \langle 2, 3, 6 \rangle$ and $\mathbf{b} = \langle 6, 2, -3 \rangle$ are two space vectors.

- Show that \mathbf{a} and \mathbf{b} are orthogonal.
- Find the scalar component of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ in the direction of the vector \mathbf{a} .
- Find a vector \mathbf{c} having as components three integers with no common factor, such that \mathbf{a} , \mathbf{b} , and \mathbf{c} form a right-handed system of mutually orthogonal vectors.
- Using your work in c), find the volume of the parallelepiped spanned by \mathbf{v} , \mathbf{a} and \mathbf{b} . (This doesn't require the calculation of a determinant!)

Problem 4 (20)

Let $A = \begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$. Its matrix of cofactors is (in part) $C = \begin{pmatrix} -2 & 2 & 4 \\ 3 & -5 & -7 \end{pmatrix}$.

- (5) Write down the missing middle row of C (be careful about the signs—if in doubt check one of the given entries in C):
- (5) The determinant of $A = -2$. Using this and part (a), find A^{-1} .
- (5) Use the previous parts to solve the system

$$x + 2y - z = 1, \quad 3x - y + 2z = 2, \quad x + y = 1.$$
- (5) In the equations in (c), if the coefficient 3 is changed to a certain number c , the equations will have no solution. Find c , without actually attempting to solve the equations.

Problem 5 (20) Let $\mathbf{r}(t) = \sin 4t\mathbf{i} + \cos 4t\mathbf{j} + 3t\mathbf{k}$.

For the helical motion in space described by the position vector $\mathbf{r}(t)$, where $t = \text{time}$, find

- the velocity vector \mathbf{v}
- the unit tangent vector \mathbf{T}
- The speed ds/dt and the arclength along the curve between the points where $t = 0$ and $t = \pi$
- The curvature k
- Show that the curve makes a constant angle with the vertical \mathbf{k} -direction

Problem 6 (10) A plane motion is given parametrically by $x = \sin t$, $y = \cos 2t$. Write the x - y equation giving the path taken by the motion, sketch the path, and describe briefly how the moving point travels along this path.

Problem 7 (10) A roll of Scotch^R tape having outer radius a is placed so its center is at the origin O , and its end P is initially at the point $(a, 0)$ on the x -axis. The end of the tape is then pulled upward, so that as it peels from the roll the peeled part is vertical. Using vector methods, give the position vector OP in terms of the parameter θ (the usual polar angle), for the values $0 \leq \theta \leq \pi$.